THEORETICAL AND EXPERIMENTAL STUDY OF PARAMETRICAL EXCITATION MICROSTRIP LATTICE WITH NONLINEAR LOADS
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ABSTRACT
In the paper the results of theoretical and experimental investigations of a microstrip lattice with nonlinear loads under parametrical excitation are discussed. For a nonlinear loaded microstrip lattice, excited plane wave, the nonlinear boundary conditions and integrated ratio in frequency domain are received for the any law of a variation in time of load parameters. A computation of reflection coefficients of a lattice is carried out for a case of harmonic variation of load parameters. Such values of amplitudes and frequencies of variation of parameters of voltage-current characteristic coefficients in case of which reflection coefficients of lattice at some combination frequencies are comparable with reflection coefficient at fundamental frequency are found theoretically. The spectrum of the signal, scattered on model of finite microstrip lattice with nonlinear loads under a parametric action on them, is experimentally studied. The features of behavior of combinational components, of harmonics of frequency of the control signal, of the reflection coefficient at the frequency of the incident wave are found experimentally when the parameters of the control signal is varied according to a harmonic law. High-quality coincidence of computing and experimental data is received.

INTRODUCTION
It is known [1] that when using the linear scatterers for whom the frequency of the re-radiated signal coincides with the frequency of primary field, there is a problem of detection of the weak signal coming from the passive scatterer, against stronger signal, obtained by re-reflection of primary field from other objects. Therefore the using of nonlinear scatterers becomes more preferable for detection of passive devices. A frequency of a detected useful signal in this case will differ from carrier frequency of primary field that will allow to get rid of noises in a reception point by means of the simple analog filter. However there is a difficulty of detecting multiple harmonics. This difficulty is caused by much more smaller field intensity at harmonic frequencies in the reception area in comparison to field at primary frequency. There is low sensitivity of receiver in the reception area [1, 2].

Besides, the parametric radiolocation of objects with the artificial NL (realizing the method of detection similar to nonlinear radar) is possible. Unlike nonlinear radar the list of found objects in a parametric radiolocation can be much wider at the expense of higher level (in comparison with harmonicas) of combinative components. The parametric radiolocation finds application, first of all, in search engines and information security systems. However known parametric scatterers are, most often, the dipole antennas loaded on the parametric oscillator [3].
With development of strict electrodynamic models of more difficult nonlinear scatterers working at combination frequencies, the list of tasks solved by a parametric radiolocation can be considerably expanded.

Really, in paper [4] the problem solution of wave reflection at combinative harmonics from the nonlinear lattice located on perfect conductive plane is considered. It is shown that reflection factors at harmonics and combination frequencies can reach essential values (at least $-15\ldots-10$ dB) in case of some values of electrophysical parameters of nonlinear loads. The considered model is the idealization far from a practical application. We will consider here structure which can have practical application.

The goal of this paper is theoretically and experimentally studies the scattering characteristics of the combination frequency components and harmonics of scattered field for the infinite microstrip lattice with non-stationary NL under the parametric excitation. The main purpose of theoretical research is the maximization of the electromagnetic (EM) field at combination frequencies. The basic idea is that the EM field at the combination frequencies can be maximized by using the selection of geometrical and electrophysical parameters of the microstrip lattice. The goal of experiment is confirmation or refutation of the results of theoretical studies for the fabricated microstrip lattice with non-stationary NL under the parametric excitation.

**THEORETICAL STUDY**

**Statement of the problem**

As structure model with NL we will consider the infinite periodic lattice of the microstrip elements in which between strips nonlinear loads can be included (Figure 1). Let VVC parameters these NL vary in time:

$$i(t) = \sum_{\nu=0}^{\infty} \left( a_{\nu}(t) u(t) + b_{\nu}(t) \frac{du(t)}{dt} \right),$$

where $i$, $u$ are current through load and voltage on load terminal pair; $a_{\nu}(t), b_{\nu}(t)$ are the coefficients determined by electrophysical properties of load.

![Figure 1: Statement of the problem](image_url)

The structure substrate of thickness $d$ represents a layer of uniform linear dielectric. We will designate internal area $-d < z < 0$ (between a ground and a strip plane) as $V_2$; external semi-infinite area ($z > 0$), filled with the homogeneous linear environment, as $V_1$.

Let a lattice be excited by the plane harmonic wave at frequency. As the researched system contains the nonlinear elements which parameters depend on time that scattered field will have the enriched frequency spectrum on comparing with a spectrum of an incident field.

Let’s system of integral equations will obtain. On a surface of the nonlinear elements included between strips, the nonlinear boundary conditions (NBC) shall be satisfied. We consider receiving NBC. Let width of the distributed nonlinear loads $\Delta x$ or $\Delta y$ (see Figure 1) is small so the electric current flows through load only in the direction, perpendicular to edges (that is or on $S_{ne x}$ sites or on $S_{ne y}$ sites). We consider that current is surface, penetration depth of a field in the direction of a vector of a normal to $S_{ne}$ site is small, and dependence of a field along the $z$ coordinate is expressed through $\delta$-function.
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Let the law of a coefficient variation in time can be provided by Fourier series at harmonics of $\omega_2$ frequency:

$$a_v = \sum_{p=-\infty}^{\infty} a_{p} e^{i\omega_2 t}; \quad b_v = \sum_{p=-\infty}^{\infty} b_{p} e^{i\omega_2 t};$$

(2)

Time functions of surface electric and magnetic currents excited on nonlinear loads will decompose in Fourier double series ($P$ is observation point):

$$\vec{j}^e(m)(P, t) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \vec{j}^e(n, k) e^{i\omega_n t};$$

(3)

where

$$\omega_{n,k} = n\omega_1 + k\omega_2.$$

On nonlinear load we express current as $i^e(P, t) = j^e(x, y, t)dx, (P \in S_{ne}x)$, voltage as $u(P, t) = \pm j^m(x, y, t)\Delta x$.

The upper sign is in case of $P \in V_1$, lower – in case of $P \in V_2$.

If the electric current flows along $y$, that $i^e(P, t) = j^e_x(x, y, t)\Delta x, \ u(P, t) = \pm j^m(y, x, t)\Delta y$.

Us transform expression (1) taking into account it, adding in it eq. (2), (3) and restricting eq. (1) by four members of a series ($Q = 3$). Operation of exponentiation is executed and orthogonality of Fourier series is considered. For $n, k$ -harmonics of an electric current we obtain:

$$\Delta y j^e_{n,k,x} = a_{k,0} \delta_{n0} \pm \Delta x \sum_{p=-\infty}^{\infty} A_{n,k-p} l^m_{n,k-p} + \Delta x^2 \sum_{p=-\infty}^{\infty} B_{n,k-p} \sum_{s_1,s_2=-\infty}^{\infty} j^m_{s_1,s_2,y} j^m_{n-s_1,k-p-s_2,y}$$

$$\mp \Delta x^3 \sum_{p=-\infty}^{\infty} C_{n,k-p} \sum_{q_1,q_2=-\infty}^{\infty} j^m_{n-q_1,k-p-q_2,y} \sum_{s_1,s_2=-\infty}^{\infty} j^m_{s_1,s_2,y} j^m_{n-q_1-s_1,k-p-q_2-s_2,y}$$

where

$$A_{n,k-p} = a_{p1} + i(n\omega_1 + (k-p)\omega_2)b_{p1}, B_{n,k-p} = a_{p2} + i(n\omega_1 + (k-p)\omega_2)b_{p2},$$

$$C_{n,k-p} = a_{p3} + i(n\omega_1 + (k-p)\omega_2)b_{p3}.$$

This expression has sense of NBC in the frequency domain in case of arbitrary variation of VCC parameters of loads.

For loads where the electric current flows along $y$ axis, NBC match with expressions (4) if in them to replace $x$ on $y$ and “$\mp$” on “$\pm$”.

**Integral ratios for fields**

Let us introduce auxiliary unit magnetic dipole at $n\omega_1 + k\omega_2$ frequency in $P$ point and orient it as $\vec{b}_{1,2} = \vec{i}_y$ or $\vec{b}_{1,2} = \vec{i}_x$ (indexes 1 and 2 mean a belonging to $V_1$ and $V_2$ areas, respectively). We select vectors of auxiliary fields $\vec{E}_{m1,2}^{n,k}, \vec{H}_{m1,2}^{n,k}$ satisfying to boundary conditions of equality of zero of $\vec{E}_{m1,2}^{n,k}$ tangent components on a surface of perfect conductor in case of $z = 0$ – for fields with an index 1, and in case of $z = 0, z = -d$ – for fields with an index 2.

A receiving of integral equations of the task is based on a use of a Lorentz lemma and boundary condition of a continuity of tangent component of vectors of a density of field on $z = 0$ boundary on sections, unoccupied by nonlinear loads, and also NBC on the surfaces of nonlinear loads. As a result, we receive the system of nonlinear integral equations (SNIE):

$$-a_{k,0} \delta_{n0} \frac{\Delta x}{\Delta y} + \Delta x \sum_{p=-\infty}^{\infty} A_{n,k-p} l^m_{n,k-p} + \Delta x^2 \sum_{p=-\infty}^{\infty} B_{n,k-p} \sum_{s_1,s_2=-\infty}^{\infty} j^m_{s_1,s_2,y} j^m_{n-s_1,k-p-s_2,y}$$

$$+ \Delta x^3 \sum_{p=-\infty}^{\infty} C_{n,k-p} \sum_{q_1,q_2=-\infty}^{\infty} j^m_{n-q_1,k-p-q_2,y} \sum_{s_1,s_2=-\infty}^{\infty} j^m_{s_1,s_2,y} j^m_{n-q_1-s_1,k-p-q_2-s_2,y} = -f^m_{n,k}$$

$$\mp \int_{S_1} j^m_{n,k} dS', P \in S_{ne}y;$$

The field, scattered by a lattice, is defined from integral ratios for scattered fields. The scattered field consists of \( \sum_{\omega} \int \psi R_{n,k}^m dS’, P \in S_{nx,y}; \)

\[
-f_{1n,k}^{m,x} = - \int_{S_1} \int_{n,k} \left( H_{n,k}^{m,1,x} + H_{n,k}^{m,2,x} \right) dS’, P \in S_1 - S_{nx,x} - S_{ny};
\]

\[
-f_{1n,k}^{m,y} = - \int_{S_1} \int_{n,k} \left( H_{n,k}^{m,1,y} + H_{n,k}^{m,2,y} \right) dS’, P \in S_1 - S_{nx,x} - S_{ny}.
\]

Here \( S_1 \) is a surface of separation of areas \( V_1 \) and \( V_2 \); the \( f_{n,k}^{m,x(y)} \) summand is defined by an incident field and isn’t equal to zero only in case of \( n = \pm 1, k = 0. \)

Using Floquet’s theorem and a formula of summing of Poisson [5], we will reduce the infinite SNIE (4) to system of nonlinear integral equations concerning unknown densities of magnetic currents on one period of a lattice.

A method of the moments in which in quality of basis function of current a ruf-top functions [6] are selected, is applied for the solution of SNIE. It also allow considering behavior of current on boundary of dielectric - a nonlinear element.

**Numerical result**

We will receive the numerical solution of the task for a case of harmonic influence at NL, varying with a frequency of \( \omega_2. \)

The field, scattered by a lattice, is defined from integral ratios for scattered fields. The scattered field consists of the plane wave reflected from the \( \varepsilon = 0 \) plane as from perfect conductive ground at \( \omega \) frequency, and a set of Floquet’s modes, their sources are the magnetic currents existing on a surface of a dielectric and nonlinear loads.

Values of currents are defined from the numerical decision of system (4).

As scattering characteristics of a lattice the reflection factors \( R_{n,k}^m \) at co-polarization at \( \omega_1+\omega_2 \) frequencies (subscript specifies number of the frequency component, superscript – number of a spatial mode) allowing to evaluate level of harmonics of scattered field in relation to an incident field at fundamental frequency are selected.

Below the numerical results, received in specific case of a two-dimensional microstrip lattice with thickness and permeability of a substrate of \( \varepsilon = 2.2, d/\lambda = 0.1 (\lambda \text{ is wavelength in free space}); \) the period of a lattice of \( d_{1,2}/\lambda = 0.1, \) with the nonlinear loads which parameters vary in time harmoniously with amplitudes of \( \omega_1 = 0.01 \text{ I/Ohm}, \omega_2 = 0.002 \text{ I/Ohm} \), are shown in the figure of normal incidence of a wave of frequency of 10 GHz.

The following designations are used: \( R_1 = |R_{1,1}|, R_2 = |R_{1,2}|, R_3 = |R_{2,1}|, R_11 = |R_{1,1}|, R_12 = |R_{2,0}|, R_{1,3} = |R_{3,1}|, R_{0,4} = |R_{0,0}|, R_{1,1} = |R_{0,0}|, R_{1,2} = |R_{2,1}|, R_{1,3} = |R_{3,1}|. \)

Calculation results of dependences of reflection factor modules from electro physical parameters of the problem and the appropriate spectra of the reflected fields (for the maximum value of the appropriate parameter), are given in Figure 2.
Energy of incident wave is transformed at frequencies of combinational harmonics of a scattered field in different proportions, and it is possible to set such value of amplitudes of VVC coefficients of nonlinear loads (for example, coefficient $a_2$) in case of which for the given frequency of parameter variation the additional harmonicas of one order or even exceed first harmonic the reflected field (Figure 2 a). And it occurs at the frequencies lying much closer to frequency of incident wave, than the multiple harmonicas.

As it is known [4], with growth of amplitude of incident wave the amplitudes of harmonicas of the reflected field grow also. In case of any parameters of the microstrip lattice, amplitudes of VVC coefficients and frequencies of their variation exists best value of $H_0$ in case of which amplitudes of additional combinative components exceed amplitude of first harmonic. Specific value of such amplitude of $H_0$ and level of exceeding of fundamental harmonic depend on the listed parameters.

**EXPERIMENTALLY STUDY**

**The prototype of the microstrip lattice with nonlinear loads**

For studying the spectrum of the scattered signal under the parametric excitation it is used a prototype of the finite microstrip lattice with nonlinear loads (Figure 3).

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**Figure 2:** Reflection coefficients of zero modes of Floquet's (a) and spectrum of the reflected field (b)
The microstrip structure (Figure 3) with non-stationary NL represents the dielectric layer of 2 mm thickness ($\varepsilon = 2.75$) and sizes of 70 mm x 70 mm. Substrate locates on the metal ground. On the surface of the dielectric are located microstrip elements (13x14 pcs.) of the square form (4 mm x 4 mm); spacing is 1 mm. Between microstrips the NL (in the form of rectifier diodes) are connected in series, that form 13 columns. Each column contains ballast resistance 33 kOhm.

The irradiation of microstrip lattice (Figure 4) is carried out at the frequency range $f_1 = 8.544 - 11.544$ GHz. Frequency of the control signal vary within the limits of the range $f_2 = 0.1 - 5$ GHz. Varying the frequency $f_2$ of control signal, we can control the spectrum of the signal scattered by microstrip lattice because the combination frequencies vary their position in the spectrum of the scattered signal.

**Results**

The task of marking of the useful signal requires the highest amplitudes of combinational components in the spectrum of the scattered signal (as far as it is possible). It is necessary also that the components are not placed too close to the fundamental frequency or to frequencies of the harmonics. Moreover, it is desirable that the maximum value of the amplitude is observed at combinational components located in the spectrum of the scattered signal lower on a frequency than harmonics of the fundamental frequency. This problem can be solved by varying the settings of the frequency of control signal $f_2$. The shape, size, physical properties of microstrip lattice and characteristics of non-stationary NL could theoretically allow to provide the amplitude of the combinational components is higher than the component amplitude at the frequency of incident wave $f_1$. It should be noted that higher amplitudes of the combinational components can be achieved through energy transfer from the multiple harmonics.

The experimental results confirmed the presence of combinational components in the spectrum of the scattered signal. Combinational components of the 1-st, 2-nd and 3-th orders are found (Figures 5 b; 6 b) when the control signal varies in time according to a harmonic law. We controlled amplitudes and frequencies of combinational components by varying the frequency and amplitude of control signal. Combinational components are located very close (Figure 5 a) to the harmonic of the probing signal and they can coincide with it. But they can’t exist in the spectrum if there is not of incident field.

During experimental studies it is found that in the spectrum of the scattered signal, in addition to the harmonics of the probing signal and to the combinational components there are harmonics of the frequency of control signal (Figure 5 a, b). The analysis shows that the harmonics of the frequency of the control signal can be not only very
close to the frequency of the probing signal, but to coincide with it (Figure 3 a). The main result is that the
reflection coefficient at the fundamental frequency changes its value when the frequency and amplitude of the
control signal are varied (Figure 6 a, b). This observation confirms that energy transfer from harmonics to
combinational components can be provided.

Figure 5: Spectra of the signal scattered by microstrip lattice with non-stationary NL (a - \( f_1=10.54 \) GHz,
\( f_2=1.5 \) GHz; b - \( f_1=11.54 \) GHz, \( f_2=1.5 \) GHz)

Harmonics of the control signal can serve as an additional tool of marking because the microstrip lattice with
control loads can radiate also on harmonics of frequency of the control signal in case of the absence of an
incident field. At the same time frequencies of these harmonics can be set up on possible frequencies of the
incident field (in a case of the absence of the most probing signal). Application of harmonics of the control signal
for marking of objects can reduce considerably the weight, the sizes and complexity of nonlinear passive detectors.
Discussion
The experimental results qualitatively confirm the computing data. They prove the principal possibility to control the combinational components in the spectrum of the scattered signal by varying the control frequency and by changing the power of external control action.

The experiment has revealed the following:
1) In the spectrum of the scattered signal in addition to the combinational components of the 1st order of \((1; 1)\) and \((1; -1)\) there are combinational components of the 2nd order of \((1; 2)\) and \((1; -2)\). Their frequencies and amplitudes can be controlled.
2) Harmonics of the frequency of control signal have high amplitudes, even higher than the amplitudes of the combinational components; and their can also be controlled.
3) Harmonic frequencies of a control signal don’t depend on the amplitude of the incident field, unlike combinational components, but they can affect the amplitude of the scattered field. They can exist in the absence of the incident field.
4) One of harmonics of the frequency of the control signal can be not only very close to the frequency of the incident field, but to coincide with it.

CONCLUSIONS
The problem solution of scattering of plane wave by the lattice of microstrip elements with parametrically varying nonlinear loads by means of a method of integral equations considerably expands an area of scientific results which can be received and analyzed by means of computing. The received results are perspective for design of the reflectors applied in the field of radio - and anti- radiolocations, nonlinear radar, electrodynamics of nonlinear structures. The experimental results qualitatively have confirmed the computing data. They have proved the principal possibility to control the combinational components in the spectrum of the scattered signal by varying the control frequency and by changing the power of external control action.

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